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Observer design with guaranteed RMS gain for discrete-time LPV systems with Markovian jumps

Giuseppe C. Calafiore[1,*,†] and Fabrizio Dabbene[2]

[1] *Dipartimento di Automatica e Informatica, Politecnico di Torino, Torino, Italy*
[2] *IEIIT-CNR, Politecnico di Torino, Torino, Italy*

SUMMARY

In this paper we consider the problem of designing state observers with guaranteed power-to-power (RMS) gain for a class of stochastic discrete-time linear systems that possess both measurable parameter variations and Markovian jumps in their dynamics. It is shown in the paper that an upper bound on the RMS gain of the observer can be characterized in terms of feasibility of a family of parameter-dependent linear matrix inequalities (LMIs). Any feasible solution to these LMIs can then be used to explicitly construct a parameter-varying jump observer that guarantees the desired performance level. This design framework is then specialized to a problem of state estimation for a linear parameter-varying plant whose state measurements are available through a lossy Bernoulli channel. Two numerical examples illustrate the results. Copyright © 2008 John Wiley & Sons, Ltd.

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KEY WORDS: observers; LPV systems; Markov jump systems; estimation over lossy channels; linear matrix inequalities

1. INTRODUCTION

Linear parameter-varying (LPV) systems are commonly used to model dynamical systems that depend on *a priori* unknown, but online measurable time-varying parameters. These models have been extensively studied in the literature, mainly due to the reason that they provide a systematic way to design gain-scheduling filters and control laws for nonlinear systems; see, for instance, [1, 2]. LPV approaches permit one to reduce the conservatism inherent in robust design methods, in cases where the *a priori* uncertain parameters can actually be acquired at the time of system

*Correspondence to: Giuseppe C. Calafiore, Dipartimento di Automatica e Informatica, Politecnico di Torino, Torino, Italy.
†E-mail: giuseppe.calafiore@polito.it

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operation; see, e.g. [3–7]. There are indeed many relevant engineering applications where this is the case. Typical examples include, for instance, altitude, speed and pressure measurements in aerospace [8], asset and configuration in manufacturing and robotics [9], concentration and pressure measurements in chemical systems [10].

On a different direction, a considerable research effort has been recently devoted to the analysis of systems whose dynamics may change abruptly and in a random manner, and in particular to the class of Markov jump linear systems [11, 12]. Such systems turn out to be particularly useful for modeling phenomena of control and estimation over networks, where random delays and packet losses may occur; see, for instance, the recent surveys [13, 14] and the references therein. The filtering problem for Markov jump systems has been extensively studied in the literature. For instance, de Souza and Fragoso [15, 16] treat the case of H_∞ filtering in continuous and discrete time, when the Markov state (mode) is available to the filter, while Feng *et al.* [17] consider the mode-independent case. If uncertainty is present in the dynamics, a robust approach for filter design is considered in [18–20]. In a similar setting, Xiong and Lam [21] analyze the case when the Markov transition matrix is uncertain. Note that in a robust approach the filter is fixed once and for all, and it gives *a priori* guaranteed performance for all possible values of the uncertainty. Thus, the robust approach is certainly useful when no information about the parameters is available during operation, but may lead to conservative designs when the *a priori* parameter uncertainty is 'large'. However, as previously discussed, in many practical situations the *a priori* unknown parameters can actually be acquired online. This information can hence be exploited by the filter in order to 'adapt' to the changes in the system dynamics and improve performance. In this 'parameter varying' setup, the filter itself should therefore be allowed to vary with the parameters. Motivated by the above considerations, this paper explores the filter design problem for Markovian jump systems in the parameter-varying framework. To the best of the authors' knowledge, this setup has not been considered in the literature yet.

More precisely, in this note we study a filtering problem for linear discrete-time systems whose dynamic matrices jump according to a finite Markov chain and are functions of *a priori* unknown but measurable parameters (J-LPV systems). For this class of systems, we study the problem of designing a mode-dependent parameter-varying observer with guaranteed power-to-power (RMS) gain between the disturbance input and the estimation error.

The key result in Section 4 provides a condition for the RMS gain to be bounded, in terms of feasibility of a set of parameter-dependent linear matrix inequalities (LMIs). When these conditions are satisfied, a jump parameter-varying observer is also explicitly constructed. In Section 4.1, this result is specialized to the case of affine dependence of parameters ranging in a polytopic domain. In this case, the convex program induced by the LMI conditions can be solved efficiently.

Note that a parameter-varying plant that is to be observed or controlled through lossy channels can be suitably modeled by a system with both parameter variations and random jumps. Therefore, in Section 5 we specialize our results to parameter-varying plants whose output measurements are available through unreliable (Bernoulli) channels (i.e. with a certain probability measurements may not be available at some instants). Two numerical examples illustrate the proposed observer design technique.

1.1. Notation

$X^\top$ denotes the transpose of matrix X; X^+ denotes the Moore–Penrose pseudoinverse of X; and $X^\perp$ denotes an orthogonal complement of X, i.e. a matrix of maximum rank such that $X^\perp X = 0$.

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I_n denotes the $n \times n$ identity matrix, and $=0_{n,m}$ denotes an $n \times m$ matrix with zero entries; subscripts with dimensions are omitted when easily inferred from context. For $X_1 \in \mathbb{R}^{n_1,n_1}$, $X_2 \in \mathbb{R}^{n_2,n_2}$, $\mathrm{diag}(X_1, X_2)$ denotes the block diagonal matrix

$$\begin{bmatrix} X_1 & 0_{n_1,n_2} \\ 0_{n_2,n_1} & X_2 \end{bmatrix}$$

$X \succ 0$ means that symmetric matrix X is positive definite. We denote by $\mathrm{co}\{\theta_1, \ldots, \theta_n\}$ the convex hull of points $\theta_1, \ldots, \theta_n$. The space of finite power sequences is denoted by $\mathscr{U}$, that is, $\mathscr{U}$ is the space of sequences z such that

$$\|z\|^2{}_{\mathrm{rms}} = \lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T} z_k^\top z_k < \infty$$

Given a sequence of random variables $\{\xi_1, \ldots, \xi_n\}$ and a function $x(\xi_1, \ldots, \xi_n)$, we denote by $\mathbb{E}_{\{\xi_1,\ldots,\xi_k\}}[x]$ the expected value of x computed with respect to $\{\xi_1, \ldots, \xi_k\}$, and by $\mathbb{E}_{\xi_{k+1}}[x \,|\, \{\xi_1, \ldots, \xi_k\}]$ the expectation of x with respect to ξ_{k+1}, conditioned on $\{\xi_1, \ldots, \xi_k\}$. The ith entry of vector x is denoted by x_i or $[x]_i$.

2. PRELIMINARIES

Consider a system $\mathscr{S}$ described by the following equations:

$$x_{k+1} = A_{\xi_k}(\theta_k) x_k + B_{\xi_k}(\theta_k) u_k \tag{1}$$

$$y_k = C_{\xi_k}(\theta_k) x_k + D_{\xi_k}(\theta_k) u_k \tag{2}$$

$$\theta_k \in \Theta, \quad \xi_k \in \{1, \ldots, N\} \tag{3}$$

where $x_k \in \mathbb{R}^n$ is the state at time k; $u_k \in \mathbb{R}^{n_u}$ is a disturbance input at time k; θ_k is a time-varying parameter that, *a priori*, is only known to belong to a given compact set $\Theta \subset \mathbb{R}^{n_\theta}$; ξ_k is a homogeneous Markov chain taking values in the finite set $\{1, \ldots, N\}$; and $y_k \in \mathbb{R}^{n_y}$ is the stochastic output of the system. The Markov chain has given transition probabilities

$$p_{i,j} \doteq \mathrm{prob}\{\xi_{k+1} = j \,|\, \xi_k = i\}, \quad i, j = 1, \ldots, N$$

The initial conditions for the system are specified by an initial state x_0 and mode ξ_0.

When useful for notational compactness, the system matrices are regrouped as

$$S_{\xi_k}(\theta_k) \doteq \begin{bmatrix} A_{\xi_k}(\theta_k) & B_{\xi_k}(\theta_k) \\ C_{\xi_k}(\theta_k) & D_{\xi_k}(\theta_k) \end{bmatrix} \tag{4}$$

The following standard notion of stability is here adopted for the stochastic system $\mathscr{S}$ see, e.g. [22].

Definition 1 (*Stochastic stability (SS)*)
Let $u_k = 0$ for $k \geqslant 0$. System (1)–(3) is said to be *stochastically stable* if for any initial conditions x_0, ξ_0 it holds that $\sum_{k=0}^{\infty} \mathbb{E}_{\Xi_k}[x_k^\top x_k] < \infty$, where $\Xi_k \doteq \{\xi_1, \ldots, \xi_k\}$.

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For purely Markov jump systems (i.e. systems possessing only the Markov parameter ξ_k, without the time-varying term θ_k), there exists a well-known necessary and sufficient characterization of stochastic stability; see, for instance, [11, 22, 23]. On the other hand, for purely parameter-varying systems (i.e. systems possessing only the θ_k parameter, without the Markovian jump structure), there exist several sufficient conditions for stability, the simplest of which (known as quadratic stability) is based on the existence of a common quadratic Lyapunov function for all possible values of $\theta_k \in \Theta$. The following lemma provides a sufficient condition for stochastic stability of the mixed parameter-varying and Markov jump system (1)–(3). This lemma extends to parameter-varying systems the stability results in [11, 22, 23]; a proof is provided in Appendix A.

Lemma 1
Suppose there exist matrices $P_i \succ 0$, $i = 1, \ldots, N$, such that

$$A_i^\top(\theta)\bar{P}_i A_i(\theta) - P_i \prec 0 \quad \forall \theta \in \Theta, \quad i = 1, \ldots, N \tag{5}$$

where $\bar{P}_i \doteq \sum_{j=1}^{N} p_{i,j} P_j$. Then, system $\mathscr{S}$ is stochastically stable.

3. A BOUND ON RMS GAIN

Assuming that $\mathscr{S}$ is stochastically stable, we say that the RMS gain of the system is less than $\gamma > 0$ if

$$\sup_{0 \neq u \in \mathscr{U}} \frac{\|y\|_{\text{rms}}}{\|u\|_{\text{rms}}} < \gamma$$

for all ξ_0 and for all y satisfying (1)–(3), with $x_0 = 0$. Here, the RMS value of the discrete-time stochastic signal y is defined as $\|y\|_{\text{rms}}{}^2 \doteq \lim_{T\to\infty} (1/T) \sum_{k=0}^{T} \mathbb{E}_{\Xi_k}[y_k^\top y_k]$. The following lemma provides a sufficient condition for a finite upper bound on the RMS gain of system $\mathscr{S}$ to exist. A proof of this result is given in Appendix A.

Lemma 2
Suppose there exist symmetric matrices $P_i \succ 0$, $i = 1, \ldots, N$ such that

$$\begin{bmatrix} \text{diag}(\bar{P}_i, I) & \text{diag}(\bar{P}_i, I) S_i(\theta) \\ * & \text{diag}(P_i, \gamma^2 I) \end{bmatrix} \succ 0 \quad \forall \theta \in \Theta, \quad i = 1, \ldots, N \tag{6}$$

where $S_i(\theta)$ is defined in (4), and $\bar{P}_i$ are defined as in Lemma 1. Then, system $\mathscr{S}$ is stochastically stable and has an RMS gain less than γ.

Remark 3
When Θ contains only one element, that is, θ_k is constant and fixed, condition (6) reduces to a known bounded real condition for Markovian jump systems, which has been proved to be both necessary and sufficient, under an additional hypothesis of weak controllability; see [24].

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4. RMS OBSERVER DESIGN

Let system (1)–(3) be given. Following the standard approach for LPV systems, we assume that the time-varying parameter θ_k can be measured online at each time instant. We assume further that the current mode ξ_k of the Markov chain is available at time k. This hypothesis is in agreement with most of the literature on Markovian jump systems see, e.g. [15, 16, 18–20]. On the basis of this information, we consider a filter having observer structure of the form

$$\hat{x}_{k+1} = A_{\xi_k}(\theta_k)\hat{x}_k + L_{\xi_k}(\theta_k)(y_k - C_{\xi_k}(\theta_k)\hat{x}_k) \tag{7}$$

where $L_{\xi_k}(\theta_k)$ is the parameter-varying filter gain, such that $L_{\xi_k}(\theta_k) = L_i(\theta_k)$ when the system is in mode $\xi_k = i$. The filtering error is defined as $e_k \doteq x_k - \hat{x}_k$. The filtering error system $\mathscr{F}$ having input u and output e is hence described by

$$e_{k+1} = \mathscr{A}_k e_k + \mathscr{B}_k u_k \tag{8}$$

with $\mathscr{A}_k \doteq A_{\xi_k}(\theta_k) - L_{\xi_k}(\theta_k)C_{\xi_k}(\theta_k)$ and $\mathscr{B}_k \doteq B_{\xi_k}(\theta_k) - L_{\xi_k}(\theta_k)D_{\xi_k}(\theta_k)$. In compact notation, the error system $\mathscr{F}$ is represented by the quadruple

$$F_{\xi_k}(\theta_k) \doteq \begin{bmatrix} \mathscr{A}_k & \mathscr{B}_k \\ I_n & 0_{n,n_u} \end{bmatrix} = \begin{bmatrix} A_{\xi_k}(\theta_k) & B_{\xi_k}(\theta_k) \\ I_n & 0_{n,n_u} \end{bmatrix} + \begin{bmatrix} -I_n \\ 0_{n,n} \end{bmatrix} L_{\xi_k}(\theta_k)[C_{\xi_k}(\theta_k) \;\; D_{\xi_k}(\theta_k)] \tag{9}$$

The following main theorem holds.

Theorem 1

Consider system $\mathscr{S}$ in (1)–(3), with $D_i(\theta)$ full row rank. Let (7) be an observer associated with system $\mathscr{S}$, and let $\gamma > 0$ be given. Define

$$\bar{P}_i \doteq \sum_{j=1}^{N} p_{i,j} P_j, \quad H_i \doteq (P_i - I_n)^{-1}, \quad R_i(\theta) \doteq D_i(\theta) D_i^\top(\theta)$$

$$N_i(\theta) \doteq \text{an orthogonal basis for } \ker D_i(\theta):\; D_i(\theta)N_i(\theta) = 0 \text{ and } N_i^\top(\theta)N_i(\theta) = I_{n_u - n_y}$$

If the following convex conditions in the variables $P_i = P_i^\top$, $i = 1, \ldots, N$:

$$P_i \succ I_n \tag{10}$$

$$\left[\begin{array}{ccc|c} \bar{P}_i & 0_{n,n} & \bar{P}_i A_i(\theta) - \bar{P}_i B_i(\theta) D_i^\top(\theta) R_i^{-1}(\theta) C_i(\theta) & \bar{P}_i B_i(\theta) N_i(\theta) \\ * & I_n & I_n & 0_{n,n_u-n_y} \\ * & * & P_i + \gamma^2 C_i^\top(\theta) R_i^{-1}(\theta) C_i(\theta) & 0_{n,n_u-n_y} \\ \hline * & * & * & \gamma^2 I_{n_u-n_y} \end{array}\right] \succ 0 \tag{11}$$

are satisfied for all $\theta \in \Theta$, $i = 1, \ldots, N$, then the observer gains

$$L_i(\theta) = \left(A_i(\theta) H_i C_i^\top(\theta) + \frac{1}{\gamma^2} B_i(\theta) D_i^\top(\theta) \right) \left(C_i(\theta) H_i C_i^\top(\theta) + \frac{1}{\gamma^2} R_i(\theta) \right)^{-1} \tag{12}$$

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$i=1,\dots,N$, guarantee that the filtering error system $\mathscr{F}$ in (9) is stochastically stable and has an RSM gain less than γ. Minimizing γ^2 subject to (10)–(11) then yields an optimized upper bound on the RMS gain of the filtering error system.

Proof
By Lemma 2, system $\mathscr{F}$ is stochastically stable and has an RMS gain less than $\gamma>0$ if there exist matrices $P_i \succ 0$, $i=1,\dots,N$, such that for all $\theta\in\Theta$, $i=1,\dots,N$

$$\begin{bmatrix} \mathrm{diag}(\bar{P}_i, I_n) & \mathrm{diag}(\bar{P}_i, I_n)F_i(\theta) \\ * & \mathrm{diag}(P_i, \gamma^2 I_{n_u}) \end{bmatrix} \succ 0 \tag{13}$$

Substituting (9) into (13), we explicitly obtain that the inequality

$$Q_i(\theta)+U_i L_i(\theta) V_i^\top(\theta) + V_i(\theta) L_i^\top(\theta) U_i^\top \succ 0 \tag{14}$$

must hold $\forall\theta\in\Theta$, and for $i=1,\dots,N$, with

$$Q_i(\theta) \doteq \begin{bmatrix} \bar{P}_i & 0_{n,n} & \bar{P}_i A_i(\theta) & \bar{P}_i B_i(\theta) \\ * & I_n & I_n & 0_{n,n_u} \\ * & * & P_i & 0_{n,n_u} \\ * & * & * & \gamma^2 I_{n_u} \end{bmatrix}, \quad U_i \doteq \begin{bmatrix} -\bar{P}_i \\ 0_{n,n} \\ 0_{n,n} \\ 0_{n_u,n} \end{bmatrix}, \quad V_i(\theta) \doteq \begin{bmatrix} 0_{n,n_y} \\ 0_{n,n_y} \\ C_i^\top(\theta) \\ D_i^\top(\theta) \end{bmatrix}$$

Consider the following orthogonal complements of U_i and $V_i(\theta)$, respectively:

$$U^\perp = \begin{bmatrix} 0_n & I_n & 0_n & 0_{n,n_u} \\ 0_n & 0_n & I_n & 0_{n,n_u} \\ 0_{n_u,n} & 0_{n_u,n} & 0_{n_u,n} & I_{n_u} \end{bmatrix}, \quad V_i^\perp(\theta) = \begin{bmatrix} I_n & 0_n & 0_n & 0_{n,n_u} \\ 0_n & I_n & 0_n & 0_{n,n_u} \\ 0_n & 0_n & I_n & -C_i^\top(\theta) D_i^{\top +}(\theta) \\ 0_{n_v,n} & 0_{n_v,n} & 0_{n_v,n} & N_i^\top(\theta) \end{bmatrix}$$

Applying to (14) the elimination lemma (Lemma A1, in Appendix A), we obtain that (14) holds for suitable gains $L_i(\theta)$, if and only if $U_i^\perp Q_i(\theta) U_i^{\perp\top} \succ 0$, $V_i^\perp(\theta) Q_i(\theta) V_i^{\perp\top}(\theta) \succ 0$. After standard matrix manipulations, it can be verified that these two conditions are equivalent to (10), (11). Equation (12) then follows from (A5), with the position

$$\Upsilon_1 = 0_{n,n_y}, \quad \Upsilon_2 = \begin{bmatrix} 0_{n,n_y} \\ C_i^\top(\theta) \\ D_i^\top(\theta) \end{bmatrix}, \quad Z_{11} = \bar{P}_i^{-1},$$

$$Z_{22} = \begin{bmatrix} I_n & I_n & 0_{n,n_u} \\ * & P_i & 0_{n,n_u} \\ * & * & \gamma^2 I_{n_u} \end{bmatrix}, \quad Z_{12}^\top = -\begin{bmatrix} 0_{n,n} \\ A_i^\top(\theta) \\ B_i^\top(\theta) \end{bmatrix} \qquad \square$$

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4.1. Polytopic parameter model

Note that, in the frequently arising case when Θ contains infinite elements, conditions (11) in Theorem 1 require the satisfaction of infinitely many LMIs, i.e. the ensuing filter design problem amounts to solving a semi-infinite convex LMI problem. Such problems are typically encountered in the LPV approach to filtering and control and may be computationally difficult to solve. In order to address this issue, different approaches are generally followed. A first approach amounts to restricting the attention to a specific class of functions of the scheduling parameters. For example, one can assume that the matrices of the LPV model are expressed as a linear fractional transformation of the underlying parameters; see, for instance, [4, 5, 25]. The original problem is then relaxed to more tractable formulae that involve a finite number of LMIs. Another classical approach is to determine an approximate solution based on finite gridding of the set Θ, see, for instance, [3, 5, 7]. Recently, approaches based on random gridding (sampling), which can deal with generic dependence on θ, have been proposed in [26, 27].

In this section we consider a special case in which the LMI conditions can be solved efficiently to any practical numerical accuracy. This special situation arises when Θ is a polytope (which encompasses the usual case of independent interval uncertainty), $A_i(\theta), B_i(\theta)$ are affine functions of θ, and $C_i(\theta), D_i(\theta)$ do not depend on θ. In this case, condition (11) is equivalent to a finite number of LMIs corresponding to the vertices of the polytope Θ. This is formally stated in the following corollary.

Corollary 1

Consider system $\mathscr{S}$ in (1)–(3), and let $D_{\xi_k}(\theta_k)=D_{\xi_k}$ be full row rank, $C_{\xi_k}(\theta_k)=C_{\xi_k}$, and

$$A_{\xi_k}(\theta_k)=A_{\xi_k}^{(0)}+\sum_{\ell=1}^{n_\theta}[\theta_k]_\ell A_{\xi_k}^{(\ell)},\quad B_{\xi_k}(\theta_k)=B_{\xi_k}^{(0)}+\sum_{\ell=1}^{n_\theta}[\theta_k]_\ell B_{\xi_k}^{(\ell)} \tag{15}$$

where

$$\theta_k\in\Theta,\quad \Theta\doteq\mathrm{co}\{\theta^{(1)},\ldots,\theta^{(m)}\}$$

Let (7) be an observer associated with system $\mathscr{S}$, and $\gamma>0$ be given. Define $\bar{P}_i, H_i, R_i, N_i$ as in Theorem 1. If the following convex conditions in the variables $P_i=P_i^\top$, $i=1,\ldots,N$:

$$P_i\succ I_n,\quad \left[\begin{array}{ccc|c} \bar{P}_i & 0_{n,n} & \bar{P}_iA_i(\theta^{(v)})-\bar{P}_iB_i(\theta^{(v)})D_i^\top R_i^{-1}C_i & \bar{P}_iB_i(\theta^{(v)})N_i\\ * & I_n & I_n & 0_{n,n_u-n_y}\\ * & * & P_i+\gamma^2C_i^\top R_i^{-1}C_i & 0_{n,n_u-n_y}\\ \hline * & * & * & \gamma^2 I_{n_u-n_y}\end{array}\right]\succ 0 \tag{16}$$

are satisfied for $v=1,\ldots,m$, $i=1,\ldots,N$, then the observer gains

$$L_i(\theta)=\left(A_i(\theta)H_iC_i^\top+\frac{1}{\gamma^2}B_i(\theta)D_i^\top\right)\left(C_iH_iC_i^\top+\frac{1}{\gamma^2}R_i\right)^{-1} \tag{17}$$

$i=1,\ldots,N$, guarantee that the filtering error system $\mathscr{F}$ in (9) is stochastically stable and has an RMS gain less than γ. Minimizing γ^2 subject to (16) yields an optimized upper bound on the RMS gain of the error system $\mathscr{F}$.

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4.2. Example 1

To illustrate the result in Corollary 1, we considered the following numerical example with two Markovian states, i.e. $\xi_k \in \{1,2\}$, and two time-varying parameters, i.e. $\theta_k \in \mathbb{R}^2$. Specifically, we choose

$$
\begin{aligned}
A_1(\theta_k) &= \begin{bmatrix} -0.5 & -0.4 \\ 0.1 & -0.8 \end{bmatrix} + [\theta_k]_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + [\theta_k]_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\
A_2(\theta_k) &= \begin{bmatrix} -0.75 & -0.5 \\ 0.1 & -0.6 \end{bmatrix} + [\theta_k]_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + [\theta_k]_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\
B_1(\theta_k) &= \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix} + [\theta_k]_1 \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix} + [\theta_k]_2 \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix} \\
B_2(\theta_k) &= B_1(\theta_k) \\
C_1 &= [-1.5 \;\; 3] \\
C_2 &= [3 \;\; -4] \\
D_1 &= D_2 = [0 \;\; 0.01]
\end{aligned}
$$

and suppose that the entries of θ_k are bounded in the interval $[-0.1,\ 0.1]$, that is,

$$
\theta_k \in \mathrm{co}\left\{ \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix}, \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}, \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \right\}
$$

Moreover, the Markov transition probabilities are set to $p_{11} = 0.9$, $p_{12} = 0.1$, $p_{21} = 0.7$, $p_{22} = 0.3$. With these data, minimizing γ^2 subject to the conditions in (16) yielded optimal upper bound on the RMS gain of the filter $\gamma = 0.2863$, and optimal parameter-varying observer gains

$$
\begin{aligned}
L_1(\theta_k) &= \begin{bmatrix} -0.5707 \\ -0.4541 \end{bmatrix} + [\theta_k]_1 \begin{bmatrix} 0.6248 \\ 0.6457 \end{bmatrix} + [\theta_k]_2 \begin{bmatrix} 0.6457 \\ 0.6248 \end{bmatrix} \\
L_2(\theta_k) &= \begin{bmatrix} -0.4262 \\ -0.0215 \end{bmatrix} + [\theta_k]_1 \begin{bmatrix} 0.4899 \\ 0.1175 \end{bmatrix} + [\theta_k]_2 \begin{bmatrix} 0.1175 \\ 0.4899 \end{bmatrix}
\end{aligned}
$$

A numerical simulation of the system (obtained for zero initial state conditions, normal random input with 0.01 variance and $[\theta_k]_1 = 0.1\sin(0.05k)$, $[\theta_k]_2 = 0.1\cos(0.05k)$) yielded the trajectories shown in Figure 1. The experimental RMS filter gain resulting from this simulation was equal to 0.116.

5. OBSERVER DESIGN WITH MISSING MEASUREMENTS

The developed framework for observer design has several applications. In particular, we next apply it in the context of systems with unreliable measurement channels, which is a key topic in networked control; see, e.g. the recent surveys [13, 14].

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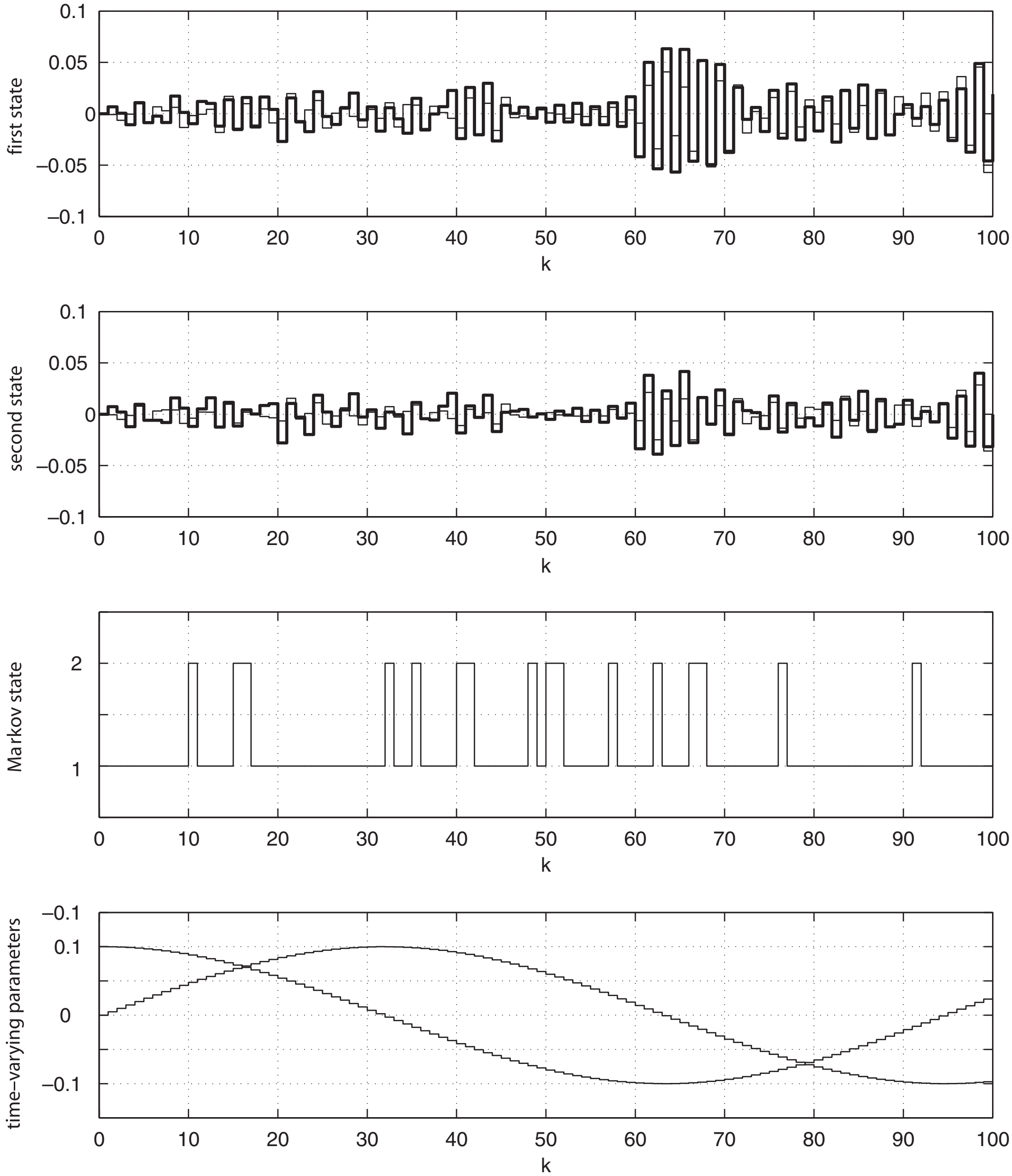

Figure 1. Simulation of system (bold) and estimator state trajectories for the system in Example 1.

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Assume that the plant is described by

$$x_{k+1} = A(\theta_k)x_k + \tilde{B}(\theta_k)v_k \tag{18}$$

and that measurements of the state are obtained through a Bernoulli channel, that is, with probability p a measurement is available according to the equation $y_k = Cx_k + \tilde{D}w_k$, and with probability $1-p$ the measure contains noise only, i.e. $y_k = \tilde{D}w_k$. This corresponds to a system having Markovian jumps in the C matrix only:

$$y_k = C_{\xi_k}x_k + \tilde{D}w_k \tag{19}$$

with $\xi_k \in \{1,2\}$ and

$$C_{\xi_k} = \begin{cases} C & \text{if } \xi_k = 1 \\ 0 & \text{if } \xi_k = 2 \end{cases}$$

The Markov chain governing the jump system is depicted in Figure 2.

This kind of lossy measurement models has been considered in several papers; see, for instance, [13, 28–30]. In the context of this note, the above situation is captured simply by taking $u_k = [v_k^\top \; w_k^\top]^\top$, $v_k \in \mathbb{R}^{n_v}$, $w_k \in \mathbb{R}^{n_w}$, and

$$B_{\xi_k}(\theta_k) = [\tilde{B}_{\xi_k}(\theta_k) \;\; 0_{n,n_w}], \quad D_{\xi_k}(\theta_k) = [0_{n_y,n_v} \;\; \tilde{D}] \tag{20}$$

The following corollary holds.

Corollary 2

Consider the LPV system (18) with unreliable measurement equation (19) governed by the Markovian model in Figure 2. Let $\tilde{D} \in \mathbb{R}^{n_y,n_w}$ be full row rank, $\Theta = \mathrm{co}\{\theta^{(1)},\ldots,\theta^{(m)}\}$, and $A_{\xi_k}(\theta_k)$, $B_{\xi_k}(\theta_k)$ be as in (15). Define

$$\bar{P} \doteq \bar{P}_1 = \bar{P}_2 = pP_1 + (1-p)P_2, \quad H \doteq (P_1 - I_n)^{-1} \tag{21}$$

If the following convex conditions in the symmetric matrix variables P_1, P_2 are satisfied for $v = 1,\ldots,m$:

$$P_1 \succ I_n, \quad P_2 \succ I_n \tag{22}$$

$$\begin{gathered}\begin{bmatrix} \bar{P} & 0_n & \bar{P}A(\theta^{(v)}) & \bar{P}[\tilde{B}(\theta^{(v)})\, 0_{n,n_w-n_y}] \\ * & I_n & I_n & 0_{n,n_u-n_y} \\ * & * & P_1 + \gamma^2 C^\top[\tilde{D}\tilde{D}^\top]^{-1}C & 0_{n,n_u-n_y} \\ * & * & * & \gamma^2 I_{n_u-n_y} \end{bmatrix} \succ 0 \\ \begin{bmatrix} \bar{P} & 0_n & \bar{P}A(\theta^{(v)}) & \bar{P}[\tilde{B}(\theta^{(v)})\, 0_{n,n_w-n_y}] \\ * & I_n & I_n & 0_{n,n_u-n_y} \\ * & * & P_2 & 0_{n,n_u-n_y} \\ * & * & * & \gamma^2 I_{n_u-n_y} \end{bmatrix} \succ 0 \end{gathered} \tag{23}$$

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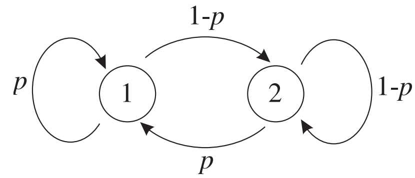

Figure 2. Markov chain for plants with missing measurements. State $\xi_1 = 1$ corresponds to an available measurement, and state $\xi_2 = 2$ corresponds to a missing measurement.

for $v = 1, \ldots, m$. Then, the observer gains

$$L_i(\theta) = \begin{cases} A(\theta) H C^\top \left(C H C^\top + \frac{1}{\gamma^2} \tilde{D}\tilde{D}^\top \right)^{-1} & \text{for } i = 1 \\ 0 & \text{for } i = 2 \end{cases}$$

guarantee that the filtering error system $\mathscr{F}$ in (9) is stochastically stable and has an RMS gain less than γ. Minimizing γ^2 subject to (22)–(23) yields an optimized upper bound on the RMS gain of the error system $\mathscr{F}$.

Proof

A proof is obtained by applying Corollary 1 to this particular special case. More precisely, define

$$\tilde{N} \in \mathbb{R}^{n_w, n_w - n_y} \doteq \text{an orthogonal basis for } \ker \tilde{D}: \ \tilde{D}\tilde{N} = 0 \quad \text{and} \quad \tilde{N}^\top \tilde{N} = I_{n_w - n_y}$$

Then, the orthogonal complements N_i defined in Corollary 1 are given by $N_1 = N_2 = \mathrm{diag}(I, \tilde{N})$. Hence, using the positions in (20), (21), with $C_1 = C$, $C_2 = 0$, and substituting these data into (16), (17), we obtain the statement. □

The previous result for the observer gain structure is in agreement with intuition. That is, when no measurement is available, the observer gain is zero, and the filter simply propagates forward the plant dynamics.

5.1. Example 2

We adapt an example originally considered in [29] in the context of Kalman filtering with intermittent observations. Consider an LPV system of the form

$$x_{k+1} = \begin{bmatrix} 1.25 & 1 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.6+\theta \end{bmatrix} x_k + \sqrt{20} v_k$$

with $v_k \in \mathbb{R}^3$. The state matrix $A(\theta)$ depends on a time-varying parameter θ, which is supposed to be measurable online and to be bounded in the polytope $\Theta = \{0, \rho\}$. When $\rho = 0$ we recover the non-varying case considered in [29]. The state of the system is measured through a Bernoulli channel,

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Table I. Value of $p_{\lim}$ for different values of ρ.

ρ	0	0.25	0.35	0.4	0.45	0.5
$p_{\lim}$	0.361	0.361	0.361	0.369	0.591	0.839

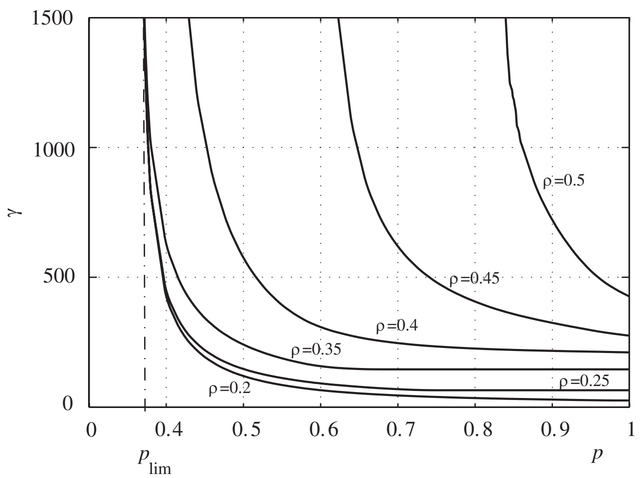

Figure 3. Plot of the RMS gain versus the probability of receiving a good measurement, for different values of the radius of parameter variation ρ.

with a probability of missing measurement equal to $1-p$. In the case of available measurement, the output equation is given by

$$y_k = [1 \;\; 0 \;\; 2]x_k + \sqrt{2.5}w_k$$

with $w_k \in \mathbb{R}$. Note that, since the considered system is unstable, we can expect that if the channel is unreliable (i.e. p is 'small') the observer will not be able to correctly follow the state trajectory, whereas if measurements arrive frequently (i.e. p is 'large') the filtering error shall be stable. It would therefore be interesting to determine the threshold value of measurement rate p below which it is no longer possible to correctly estimate the states of the system. Specifically, we can compute an upper bound $p_{\lim}$ on the minimum value of p such that the filtering error system $\mathscr{F}$ is stable. That is, for fixed ρ, we consider the minimization problem

$$\underset{\gamma^2, p, P_1, P_2}{\text{minimize}}\; p \quad \text{subject to } p \in (0,1) \text{ and (22)–(23)} \tag{24}$$

This problem can be easily solved by bisection over p, where each step in the bisection method requires checking feasibility of the LMIs considered in Corollary 2. For $\rho = 0$, we obtain the value of $p_{\lim} = 0.361$, which numerically coincides with the one obtained in [29]. Then, we run the optimization (24) for increasing values of the uncertainty radius ρ. The results are reported in Table I.

In Figure 3 we also report the values of the RMS gain γ obtained optimizing γ^2 over (22)–(23) for different values of ρ and values of p ranging in $(p_{\lim}, 1)$.

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We note, as expected, that an increase in the radius of uncertainty ρ corresponds to a degradation in the performance of the filter. Moreover, in our experiments, we observed that an abrupt change in the limit value in the admissible level of measurement rate $p_{\lim}$ occurs when ρ reaches the value 0.4. This type of behavior has also been observed in [29] and corresponds to the situation where the system matrix $A(\theta)$ may have more than one unstable eigenvalue.

6. CONCLUSIONS

This paper discussed an RMS filtering problem for discrete-time systems that present both parameter variations and Markovian jumps in their system matrices. The key result in Theorem 1 provides LMI conditions for guaranteeing that the filtering error system is stable and has (squared) RMS gain less than a given level γ^2. An optimized filter can then be obtained by minimizing the level γ^2 subject to these conditions. The resulting convex optimization problem can be solved exactly when Θ is of finite cardinality, or approximately via deterministic or probabilistic gridding techniques, otherwise. In the particular case of polytopic LPV parameters, the problem becomes a standard convex LMI optimization problem, which can be solved in polynomial time. Models of LPV systems whose measurements are available through lossy channels fit into the considered class and have been analyzed as a special case in Section 5.

APPENDIX A

Proof of Lemma 1

We start with a preliminary technical result: System $\mathscr{S}$ is stochastically stable if there exists a stochastic Lyapunov function $V(k) \doteq V(x_k, \xi_k) = x_k^\top P_{\xi_k} x_k$, with $P_{\xi_k} = P_i \succ 0$ when $\xi_k = i$, such that for all x_k satisfying the system equations (1)–(3) it holds that $\mathbb{E}_{\Xi_{k+1}}[\Delta V(k)] < 0$, where $\Delta V(k) \doteq V(k+1) - V(k)$.

To prove this statement, note that, since $\mathbb{E}_{\Xi_{k+1}}[\Delta V(k)] < 0$ and $V(k) > 0$, there exists $\varepsilon \in (0,1)$ such that $\mathbb{E}_{\Xi_{k+1}}[\Delta V(k)] = \mathbb{E}_{\Xi_{k+1}}[V(k+1)] - \mathbb{E}_{\Xi_k}[V(k)] \leqslant -\varepsilon \mathbb{E}_{\Xi_k}[V(k)]$; hence, $\mathbb{E}_{\Xi_{k+1}}[V(k+1)] \leqslant (1-\varepsilon)\mathbb{E}_{\Xi_k}[V(k)]$. Therefore, $\mathbb{E}_{\Xi_k}[V(k)] \leqslant (1-\varepsilon)^k V(0)$. Summing over k from 0 to infinity, we get $\sum_{k=0}^{\infty} \mathbb{E}_{\Xi_k}[V(k)] = \sum_{k=0}^{\infty} \mathbb{E}_{\Xi_k}[x_k^\top P_{\xi_k} x_k] \leqslant (1/\varepsilon) V(0)$. Since $P_{\xi_k} \succ 0$, this implies that $\sum_{k=0}^{\infty} \mathbb{E}_{\Xi_k}[x_k^\top x_k] < \infty$, which proves the preliminary statement.

Suppose now (5) holds and define the stochastic Lyapunov function $V(k) = x_k^\top P_{\xi_k} x_k$ with $P_{\xi_k} = P_i$, for $\xi_k = i$. Then, we have $\mathbb{E}_{\Xi_{k+1}}[\Delta V(k)] = \mathbb{E}_{\Xi_k}[\mathbb{E}_{\xi_{k+1}}[\Delta V(k)|\Xi_k]] = \mathbb{E}_{\Xi_k}[\mathbb{E}_{\xi_{k+1}}[V(k+1)|\Xi_k] - V(k)]$, where the last equality follows from the fact that $V(k)$ does not depend on ξ_{k+1}. Note now that $\mathbb{E}_{\xi_{k+1}}[V(k+1)|\Xi_k] = \mathbb{E}_{\xi_{k+1}}[x_{k+1}^\top P_{\xi_{k+1}} x_{k+1}|\Xi_k] = \mathbb{E}_{\xi_{k+1}}[x_k^\top A_{\xi_k}(\theta_k)^\top P_{\xi_{k+1}} A_{\xi_k}(\theta_k) x_k|\Xi_k] = x_k^\top A_{\xi_k}(\theta_k) \mathbb{E}_{\xi_{k+1}}[P_{\xi_{k+1}}|\Xi_k] A_{\xi_k}(\theta_k) x_k$. From the Markov property, it then follows that

$$\mathbb{E}_{\xi_{k+1}}[P_{\xi_{k+1}}|\Xi_k] = \mathbb{E}_{\xi_{k+1}}[P_{\xi_{k+1}}|\xi_k] = \sum_{j=1}^{N} p_{\xi_k, j} P_{\xi_k} = \bar{P}_{\xi_k}$$

hence $\mathbb{E}_{\Xi_{k+1}}[\Delta V(k)] = \mathbb{E}_{\Xi_k}[x_k^\top (A_{\xi_k}(\theta_k)^\top \bar{P}_{\xi_k} A_{\xi_k}(\theta_k) - P_{\xi_k}) x_k]$. It follows from (5) that this latter expression is negative for all x_k satisfying the system equations; therefore, stochastic stability of $\mathscr{S}$ follows from the preliminary result. □

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Proof of Lemma 2

Again, we start by stating a preliminary result: Suppose there exists a stochastic Lyapunov function $V(k)=V(x_k,\xi_k)=x_k^\top P_{\xi_k}x_k$, with $P_{\xi_k}=P_i\succ 0$ for $\xi_k=i$, such that

$$\mathbb{E}_{\Xi_{k+1}}[\Delta V(k)]<\gamma^2 u_k^\top u_k-\mathbb{E}_{\Xi_k}[y_k^\top y_k] \tag{A1}$$

holds for all $u\in\mathscr{U}$, for all x_k, y_k satisfying (1)–(3), and for all $k\geqslant 0$. Then, system $\mathscr{S}$ is stochastically stable and has an RMS gain less than $\gamma>0$.

To prove this preliminary statement, note that since (A1) must hold for all $u\in\mathscr{U}$, choosing $u=0$, we have $\mathbb{E}_{\Xi_{k+1}}[\Delta V(k)]<0$, which implies stochastic stability. Note next that

$$\sum_{k=0}^{T}\mathbb{E}_{\Xi_{k+1}}[\Delta V(k)]=\mathbb{E}_{\Xi_{T+1}}[V(T+1)]-V(0)=\mathbb{E}_{\Xi_{T+1}}[V(T+1)]$$

where we used the fact that $x_0=0$ when $V(0)=0$. Therefore, summing over (A1) for $k=0$ to T, we get $\mathbb{E}_{\Xi_{T+1}}[V(T+1)]<\gamma^2\sum_{k=0}^{T}u_k^\top u_k-\sum_{k=0}^{T}\mathbb{E}_{\Xi_k}[y_k^\top y_k]$, $\forall T\geqslant 0$. Since $\mathbb{E}_{\Xi_{T+1}}[V(T+1)]\geqslant 0$, it follows that $\sum_{k=0}^{T}\mathbb{E}_{\Xi_k}[y_k^\top y_k]<\gamma^2\sum_{k=0}^{T}u_k^\top u_k$, $\forall T\geqslant 0$. Hence, dividing both sides by T and taking the limit for $T\to\infty$, we get that $\|y\|_{\text{rms}}{}^2<\gamma^2\|u\|_{\text{rms}}{}^2$ holds for all $u\in\mathscr{U}$ and y_k satisfying (1)–(3), which proves the preliminary statement.

Suppose now that (6) holds and define $V(k)=x_k^\top P_{\xi_k}x_k$ with $P_{\xi_k}=P_i$ for $\xi_k=i$. We have

$$\begin{aligned}
&\mathbb{E}_{\Xi_k}[\mathbb{E}_{\xi_{k+1}}[V(k+1)\,|\,\Xi_k]-V(k)]\\
&\quad=\mathbb{E}_{\Xi_k}[\mathbb{E}_{\xi_{k+1}}[(A_{\xi_k}(\theta_k)x_k+B_{\xi_k}(\theta_k)u_k)^\top P_{\xi_{k+1}}(A_{\xi_k}(\theta_k)x_k+B_{\xi_k}(\theta_k)u_k)\,|\,\Xi_k]-x_k^\top P_{\xi_k}x_k]\\
&\quad=\mathbb{E}_{\Xi_k}[(A_{\xi_k}(\theta_k)x_k+B_{\xi_k}(\theta_k)u_k)^\top \bar{P}_{\xi_k}(A_{\xi_k}(\theta_k)x_k+B_{\xi_k}(\theta_k)u_k)-x_k^\top P_{\xi_k}x_k]\\
&\quad<\gamma^2u_k^\top u_k-\mathbb{E}_{\Xi_k}[(C_{\xi_k}(\theta_k)x_k+D_{\xi_k}(\theta_k)u_k)^\top(C_{\xi_k}(\theta_k)x_k+D_{\xi_k}(\theta_k)u_k)]
\end{aligned}$$

The latter expression may be rewritten as

$$\mathbb{E}_{\Xi_k}\begin{bmatrix}x_k\\u_k\end{bmatrix}^\top\begin{bmatrix}A_{\xi_k}(\theta_k)^\top\bar{P}_{\xi_k}A_{\xi_k}(\theta_k)-P_{\xi_k}+C_{\xi_k}(\theta_k)^\top C_{\xi_k}(\theta_k) & A_{\xi_k}(\theta_k)^\top\bar{P}_{\xi_k}B_{\xi_k}(\theta_k)+C_{\xi_k}(\theta_k)^\top D_{\xi_k}(\theta_k)\\ * & B_{\xi_k}(\theta_k)^\top\bar{P}_{\xi_k}B_{\xi_k}(\theta_k)+D_{\xi_k}(\theta_k)^\top D_{\xi_k}(\theta_k)-\gamma^2 I\end{bmatrix}\begin{bmatrix}x_k\\u_k\end{bmatrix}<0$$

which is satisfied for all x_k, u_k that satisfy the system equations if the following LMI holds for all $\theta\in\Theta$, $i=1,\ldots,N$:

$$\begin{bmatrix}-A_i^\top(\theta)\bar{P}_iA_i(\theta)+P_i-C_i^\top(\theta)C_i(\theta) & -A_i^\top(\theta)\bar{P}_iB_i(\theta)-C_i^\top(\theta)D_i(\theta)\\ * & -B_i^\top(\theta)\bar{P}_iB_i(\theta)-D_i^\top(\theta)D_i(\theta)+\gamma^2 I\end{bmatrix}\succ 0$$

Let $\Omega_i\doteq\text{diag}(P_i,\gamma^2 I)$, $\bar{\Omega}_i\doteq\text{diag}(\bar{P}_i, I)$ and

$$S_i(\theta)\doteq\begin{bmatrix}A_i(\theta) & B_i(\theta)\\ C_i(\theta) & D_i(\theta)\end{bmatrix}$$

Then the left-hand side of the previous inequality is rewritten as

$$\Omega_i-S_i^\top(\theta)\bar{\Omega}_iS_i(\theta)=\Omega_i-S_i^\top(\theta)\bar{\Omega}_i\bar{\Omega}_i^{-1}\bar{\Omega}_iS_i(\theta)$$

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Hence, applying the Schur complement rule to the inequality, we obtain

$$\begin{bmatrix} \bar{\Omega}_i & \bar{\Omega}_i S_i(\theta) \\ S_i^\top(\theta)\bar{\Omega}_i & \Omega_i \end{bmatrix} \succ 0$$

which is the statement in (6). □

Lemma A1 (*Elimination, see* [*31*])
The matrix inequality

$$Q + UYV^\top + VY^\top U^\top \succ 0 \tag{A2}$$

holds for some Y if and only if

$$U^\perp Q U^{\perp\top} \succ 0 \quad \text{or} \quad UU^\top \succ 0 \tag{A3}$$

$$V^\perp Q V^{\perp\top} \succ 0 \quad \text{or} \quad VV^\top \succ 0 \tag{A4}$$

where $U^\perp, V^\perp$ are the orthogonal complements of U, V, respectively. Furthermore, if U, V are full column rank and (A3), (A4) are satisfied, then a matrix Y that satisfies (A2) is given by

$$Y = (\Upsilon_1 - Z_{12} Z_{22}^{-1} \Upsilon_2)(\Upsilon_2^\top Z_{22}^{-1} \Upsilon_2)^{-1} \tag{A5}$$

where

$$\begin{bmatrix} \Upsilon_1 \\ \Upsilon_2 \end{bmatrix} \doteq \begin{bmatrix} U^+ \\ U^\perp \end{bmatrix} V, \quad \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^\top & Z_{22} \end{bmatrix} \doteq \begin{bmatrix} U^+ \\ U^\perp \end{bmatrix} Q \begin{bmatrix} U^+ \\ U^\perp \end{bmatrix}^\top$$

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